HA Models from FRB Economists and Life-Cycle Jacobians

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Research in Central Banks, Where do HA Models Fit?

Waller (2025):

- Central banks often use models to think through emerging shocks or scenarios.
- Research building knowledge and models ahead of time is useful.

In recent years:

- Growing mass of FRB economists using HA models in research.
- SSJ method/framework has facilitated adoption and collaboration.

Today:

- Two HANK models from working papers by FRB economists. (far from exhaustive!)
- Examples of shocks/scenarios for which similar models *might* be relevant.
- Thoughts and code for using SSJ with life cycle/OLG models.

The Macroeconomic Effects of Excess Savings,
Bardoczy, Sim, and Tischbirek (2024)

Post-Pandemic Savings

Reduced consumption + Govt. transfers = "Excess" savings ($\approx$ 10% Pre-COVID GDP)

Attention from the media...

Covid-19 Savings Stockpile Could Accelerate Economy—if Consumers Spend It

Households amassed an unprecedented pile of savings as the pandemic crimped their ability to spend, but it is unclear how much will be used as the economy reopens since a lot of it is held

By *Paul Hannon*

Americans Can't Stop Spending. Five Reasons Why.

A mix of job security, high savings and skepticism about the future a

By *Rachel Wolfe* and *Amara Omeokwe*

Americans Tap Pandemic Savings to Cope With Inflation

With wage gains lagging behind soaring inflation, U.S. households are star accumulated over the first two years of the pandemic.

By *Rachel Louise Ensign* and *Orla McCaffrey*

...and from Federal Reserve economists

(Batty, Deeken, and Volz 2021; Abdelrahman and Oliveira 2023; Aladangady et al. 2022; DeLuca and Pinheiro 2023)

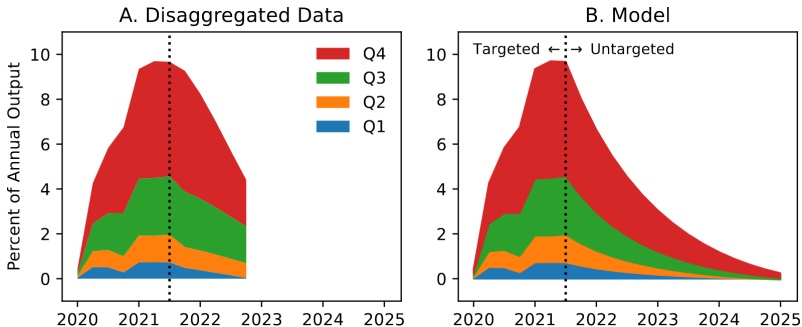
Main questions: when will they be spent?

- Stark answers in RANK and TANK (Rognlie 2024).

Model Fit and Predictions

- Model: HANK calibrated to iMPCs and fiscal multipliers.
  Ramey and Zubairy (2018) and Fagereng, Holm, and Natvik (2021)

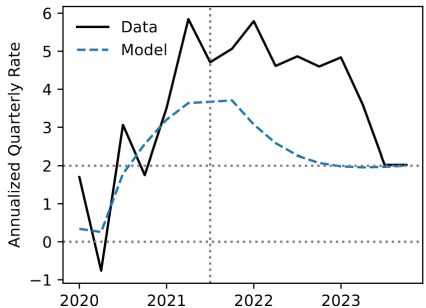
Excess savings by income quartiles (data from Aladangady et al. 2022)



Model matches gradual depletion! iMPCs characterize "excess savings"

Model Fit and Predictions

Core PCE inflation: model accounts for $\approx 40\%$ of observed dynamics



Depends on slope of Philips curve

- Calibrate to Gagliardone et al. (2023).
- Sensitivity exercises in the paper.

HANK Comes of Age,
Bardóczy and Velásquez-Giraldo (2024)

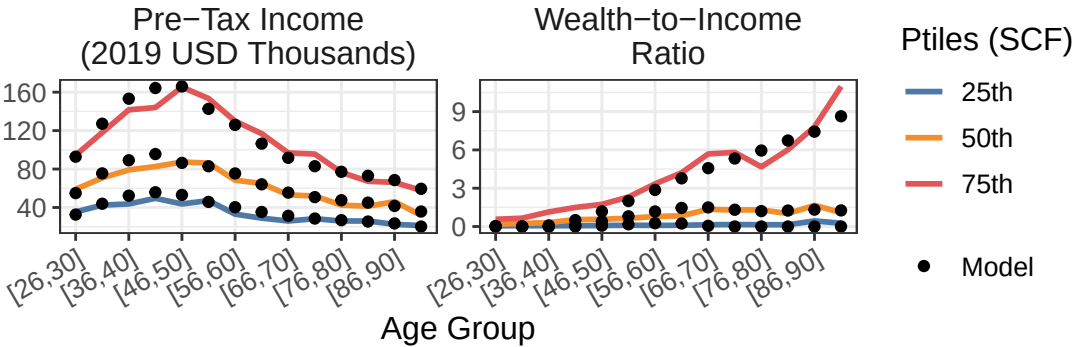
Life Cycle Structure in a HANK Model... But Why?

- Most important **H**'s in **H**ANK literature change as people age.
  MPCs, wealth, portfolios, exposure to macro fluctuations, ...
  - Forces like retirement savings could change aggregate dynamics.
    (e.g., Beaudry, Cavallino, and Willems 2024)
  - Understand transmission channels/state dependence.
- Demographics featuring more prominently in policy discussions.
  Auclert et al. (2021), Platzer and Peruffo (2022), and Gruss et al. (2025)
- Many potential questions/scenarios:
  - Monetary policy in an aging population.
  - Effects of reduction to old-age entitlements.
  - Effects of immigration.
  - ...

Wealth and Income Profiles (SCF 2019)

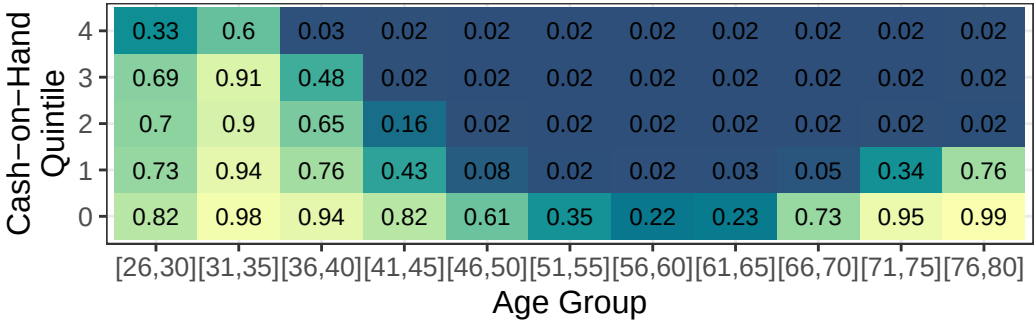
▸ Uncond. Wealth Dist.

- Ages 26 to 100, annual.
- Income process from Arellano, Blundell, and Bonhomme (2017).
- "Luxury" bequest motive (Carroll 2002; De Nardi 2004).



* *"Wealth" = Financial assets*

Marginal Propensities to Consume



- **Average annual MPC: 0.41**, while matching financial assets.
- MPC co-varies with age and wealth in expected directions. (Fagereng, Holm, and Natvik 2021)
- 34% of households are hand-to-mouth (MPC≈1). (Aguiar, Bils, and Boar 2020; McKay and Wolf 2023)

Expansionary MP Shock: Who Responds and Why?

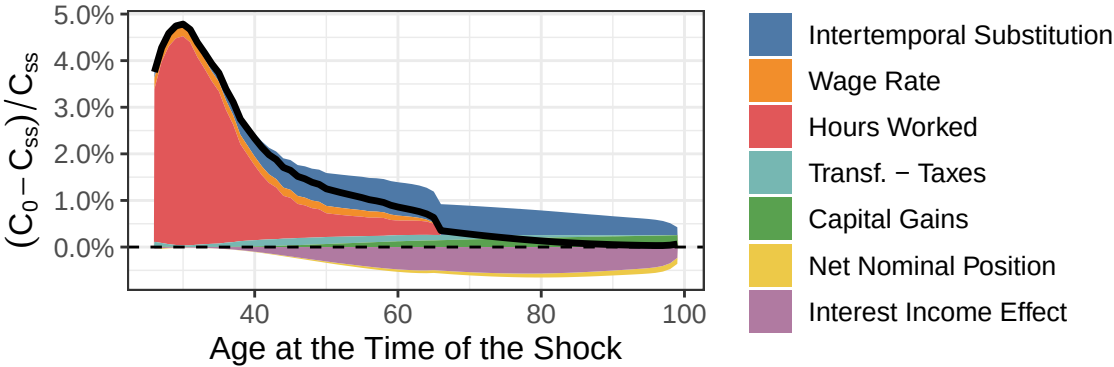
Embed household block in a New Keynesian environment

- Sticky prices, sticky wages, fiscal and monetary bloks,...

Study monetary expansion: $R_t^n = R_{ss}^n + \phi_\pi(\pi_t - \pi_{ss}) + \varepsilon_{\mathbf{t}}$

- Shock: $\{\varepsilon_{\mathbf{t}}\}_{t\geq 0} = \{ -0.01 \times 0.5^t\}_{t\geq 0}$

Expansionary MP Shock: Who Responds and Why?



- **Young**: disposable income from higher labor demand + high MPCs.
- **Middle Aged**: retirement saving decision starts to matter (substitution vs income).
- **Retirees**: asset returns, income un-does positive effects.

Sequence-Space Jacobians of Life Cycle Models,
Bardoczy and Velasquez Giraldo (2025)

Representing Life Cycle Models in SJJ Framework

Life cycle model:

- Ages from 0 to $A-1$.
- Exogenous death (δ_a).
- Newborns draw states from η.

Partition SSJ objects into age-specific components:

$$\mathbf{v}'_t = \left[\mathbf{v}'_t(0), ..., \mathbf{v}'_t(A-1)\right], \quad \mathbf{D}'_t = \left[\mathbf{D}'_t(0), ..., \mathbf{D}'_t(A-1)\right], \quad \mathbf{y}'_t = \left[\mathbf{y}'_t(0), ..., \mathbf{y}'_t(A-1)\right]$$

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$$\mathbf{\Lambda}_t = \begin{bmatrix} \delta_0 \times \mathbf{1}\eta' & \not{\delta}_0 \mathcal{L}_t(0) & \mathbf{0} & ... & \mathbf{0} \\ \delta_1 \times \mathbf{1}\eta' & \mathbf{0} & \not{\delta}_1 \mathcal{L}_t(1) & ... & \mathbf{0} \\ \vdots & \vdots & \vdots & ... & \vdots \\ \delta_{A-2} \times \mathbf{1}\eta' & \mathbf{0} & \mathbf{0} & ... & \not{\delta}_{A-2} \mathcal{L}_t(A-2) \\ \mathbf{1}\eta' & \mathbf{0} & \mathbf{0} & ... & \mathbf{0} \end{bmatrix}$$

Life Cycle Analogue of Fake News Algorithm

Age-specific Jacobians $\{\mathcal{J}(a)\}_{a=0}^{A-1}$:

$$\mathcal{J}_{t,s}(a) \equiv \text{Part of } \mathcal{J}_{t,s} \text{ due to agents of age } a \text{ at time } t$$

Age-specific Fake News Matrices $\{\mathcal{F}(a)\}_{a=0}^{A-1}$:

$$\mathcal{F}_{t,s}(a) \equiv \text{Part of } \mathcal{F}_{t,s} \text{ due to agents of age } a \text{ at time } t$$

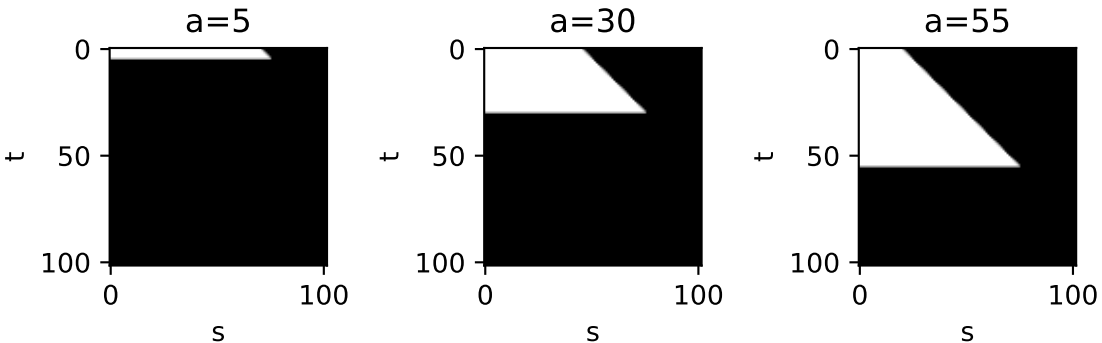
Aggregates: $\mathcal{J}_{t,s} = \sum_{a=0}^{A-1} \mathcal{J}_{t,s}(a)$, $\mathcal{F}_{t,s} = \sum_{a=0}^{A-1} \mathcal{F}_{t,s}(a)$.

All very similar to aggregate Fake News algorithm, but **there are many 0s!**

Life Cycle Analogue of Fake News Algorithm

Agents relevant to $\mathcal{F}_{t,s}(a)$ are aged $a - t$ when fake news arrive (time 0).

$$\mathcal{F}_{t,s}(a) \neq 0 \textbf{ only if } \underbrace{0 \leq a - t}_{\text{Had been born}} \text{ and } \underbrace{a - t + s \leq A - 1}_{\text{Not past terminal age at time } s}$$



Cookbook: Bardoczy and Velasquez Giraldo (2025)

Transform an infinite horizon model to an OLG with A ages.

How costly is it to get H-horizon Jacobians ($H > A$)?

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How costly is it to get H-horizon Jacobians ($H > A$)?

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	Infinite Horizon	Life-Cycle (75 Ages)	Ratio (LC/IH)
No. of idiosyncratic states	357	26,775	75
Jacobian ($H = 300$) calc. time	0.12 s	0.89 s	7.32

[Paper]: shows how to get Jacobians from a basic life-cycle solver.

[Code] is public!

Thank you!

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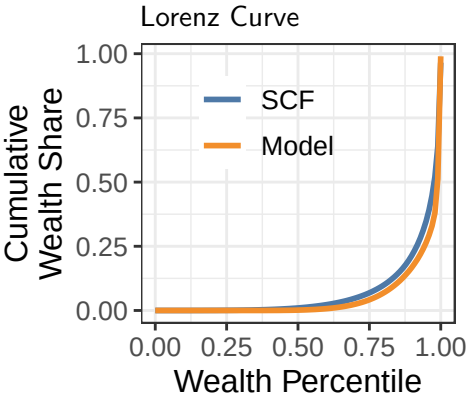
Unconditional Wealth Distribution

Back



- Approximates wealth inequality well.
- Gini Coeff: 0.9 (0.86 in SCF data).
- Wealth more unequal than income (Gini 0.61).
- Difficult in standard HANK.
- Main contributors in our model:
  - Gradual build-up of wealth.
  - Income process, persistent high earners.
  - Luxury bequests.